Answer ALL questions. Write your answers in the spaces provided.

1. The line l passes through the points A (3, 1) and B (4, −2).

Find an equation for l.

(3)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2-1}{4-3}$$

$$= -3$$

Equation: $y - y_1 = m(x - x_1)$

Using $A(3,1)$ and $m = -3$ → $y - 1 = -3(x-3)$

$$y - 1 = -3x + 9$$

$$\boxed{y = -3x + 10}$$

(Total for Question 1 is 3 marks)

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2. The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6).

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

$y = 2x^2 - 12x + 16$

$\frac{dy}{dx} = 4x - 12$

when $x = 5$, $\frac{dy}{dx} = 4(5) - 12$

$= 20 - 12$

$\boxed{= 8}$

$\therefore$ the gradient at $P(5,6)$ is 8

(Total for Question 2 is 4 marks)

3. Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

(a) $\overrightarrow{AB} = (8-3)i + (3-(-7))j$

$\boxed{\overrightarrow{AB} = 5i + 10j}$

(b) $|\overrightarrow{AB}|$ = length of vector $\overrightarrow{AB}$

$= \sqrt{5^2 + 10^2}$

$= \sqrt{25+100}$

$= \sqrt{125}$

$= \sqrt{25 \times 5}$

$\boxed{= 5\sqrt{5}}$

(Total for Question 3 is 4 marks)

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4.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(a) If $(x-3)$ is a factor of $f(x)$, then $f(3)=0$

$f(3) = 4(3)^3 - 12(3)^2 + 2(3) - 6$

$= 108 - 108 + 6 - 6$

$= 0$

$\therefore$ $(x-3)$ is a factor of $f(x)$.

(b) $4x^3 - 12x^2 + 2x - 6 = (x-3)(4x^2+2)$

For $4x^2+2=0$, $a=4$, $b=0$, $c=2$

Discriminant, $b^2 - 4ac = 0^2 - (4)(4)(2)$

$= -32$

Since the discriminant is less than 0, $4x^2+2=0$ has no real roots.

$\therefore$ 3 is the only real root of the equation $f(x)=0$

(Total for Question 4 is 6 marks)

5. Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

$f(x) = 2x + 3 + 12x^{-2}$

$$\int_1^{2\sqrt{2}} \left(2x + 3 + 12x^{-2}\right) dx = \left[\frac{2x^2}{2} + 3x + \frac{12x^{-1}}{-1}\right]_1^{2\sqrt{2}}$$

$$= \left[x^2 + 3x - \frac{12}{x}\right]_1^{2\sqrt{2}}$$

$$= \left((2\sqrt{2})^2 + 3(2\sqrt{2}) - \frac{12}{2\sqrt{2}}\right) - \left(1^2 + 3(1) - \frac{12}{1}\right)$$

$$= \left(8 + 6\sqrt{2} - \frac{6}{\sqrt{2}}\right) - \left(1 + 3 - 12\right)$$

$$= \left(8 + 6\sqrt{2} - \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}}\right) - (-8)$$

$$= \left(8 + 6\sqrt{2} - \frac{6\sqrt{2}}{2}\right) + 8$$

$$= 8 + 6\sqrt{2} - 3\sqrt{2} + 8$$

$$= \boxed{16 + 3\sqrt{2}}$$

(Total for Question 5 is 5 marks)

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6. Prove, from first principles, that the derivative of $3x^2$ is $6x$.

(4)

Let $f(x) = 3x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + hx + hx + h^2) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6hx + 3h^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{6hx + 3h^2}{h}$$

$$= \lim_{h \to 0} 6x + 3h$$

As $h \to 0$, $3h \to 0$, so $f'(x) \to 6x$

So in the limit, $\boxed{\text{derivative} = 6x}$

(Total for Question 6 is 4 marks)

7. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2 - \frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7

(1)

(a) $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$

From formula booklet

So, $\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6 \cdot \left(-\frac{x}{2}\right) + \binom{7}{2}2^5 \cdot \left(-\frac{x}{2}\right)^2 + \ldots$

$= 128 + (7)(64)\left(-\frac{x}{2}\right) + (21)(32)\left(\frac{x^2}{4}\right) + \ldots$

$= 128 + 448\left(-\frac{x}{2}\right) + 672\left(\frac{x^2}{4}\right) + \ldots$

$= 128 - \frac{448x}{2} + \frac{672x^2}{4} + \ldots$

$= \boxed{128 - 224x + 168x^2 + \ldots}$

(b) $\left(2 - \frac{x}{2}\right) = 1.995$

$\frac{x}{2} = 0.005$

$\underline{x = 0.01}$

To estimate a value for 1.995^7, you would substitute 0.01 for x into the expansion

8.

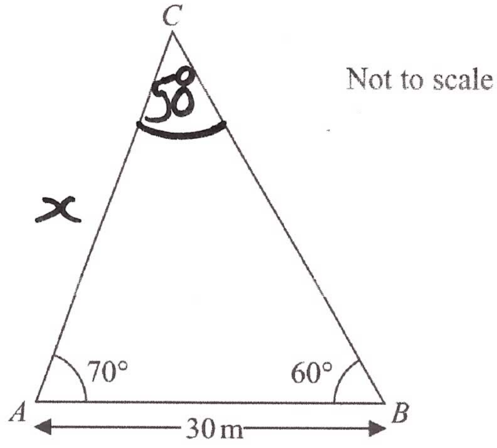

Figure 1

A triangular lawn is modelled by the triangle ABC, shown in Figure 1. The length AB is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

(4)

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

(a) Angle $A\hat{C}B = 50°$ (angles in a Δ sum to 180°)

By sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

So, $\dfrac{x}{\sin 60} = \dfrac{30}{\sin 50}$

$x = \dfrac{30}{\sin 50} \times \sin 60$

$x = \dfrac{30 \sin 60}{\sin 50}$

$x = 33.9\text{m}$ (to 3s.f.)

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Question 8 continued

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(30)(33.9)(\sin 70)$

$= 478.05\ldots$

$= \boxed{478\text{ m}^2 \text{ (to 3s.f.)}}$

(b) It is unlikely to be accurate to the nearest square metre because it is unlikely that the lawn is exactly flat, so modelling by a plane figure may not be accurate.

(Total for Question 8 is 5 marks)

9. Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

Since $\sin^2 x \equiv 1 - \cos^2 x$,

$$12(1-\cos^2 x) + 7\cos x - 13 = 0$$

$$12 - 12\cos^2 x + 7\cos x - 13 = 0$$

$$-1 - 12\cos^2 x + 7\cos x = 0$$

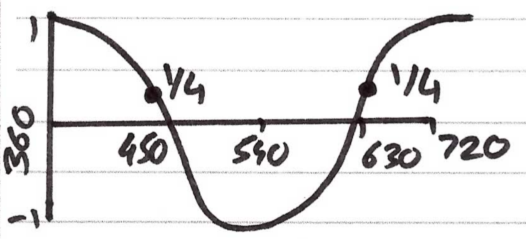
$$12\cos^2 x - 7\cos x + 1 = 0$$

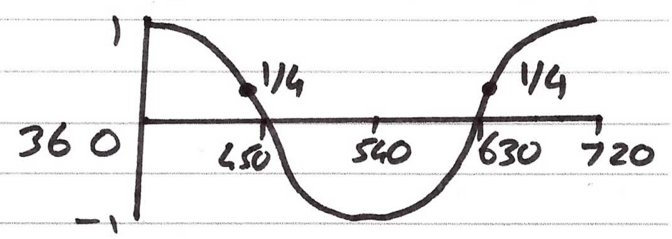
$$(4\cos x - 1)(3\cos x - 1) = 0$$

Either $4\cos x - 1 = 0$ or $3\cos x - 1 = 0$

$4\cos x = 1$ $\quad$ $3\cos x = 1$

$\cos x = \frac{1}{4}$ $\quad$ $\cos x = \frac{1}{3}$





$\cos^{-}(\frac{1}{4}) = 75.5°$ $\quad$ $\cos^{-1}(\frac{1}{3}) = 70.5°$

In given domain, $x = 360 + 75.5$ or $360 + 70.5°$

$$\boxed{x = 430.5°,\ 435.5° \text{ (to 1d.p.)}}$$

10. The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

If $k=0$, then $0x^2+4(0)x+3=0$

$3=0$

which gives no real roots

Discriminant, $b^2-4ac = (4k)^2-(4)(k)(3)$

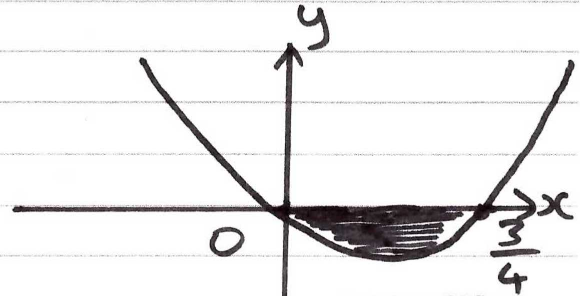
So, $(4k)^2 -(4)(k)(3) < 0$

$16k^2 - 12k < 0$

For $16k^2 - 12k = 0$

$4k(4k-3) = 0$

Either $k=0$ or $k=\frac{3}{4}$



Choose the values 'under' the x-axis, since $16k^2-12k<0$

$0<k<\frac{3}{4}$

Together with $k=0$ giving an unreal solution,

$$\boxed{0 \leq k < \frac{3}{4}}$$

(Total for Question 10 is 4 marks)

11. (a) Prove that for all positive values of x and y

$$\sqrt{xy} \leqslant \frac{x+y}{2}$$

(2)

(b) Prove by counter example that this is not true when x and y are both negative.

(1)

(a) Since x and y are both positive, their square roots are real, and so we can use:

$$(\sqrt{x} - \sqrt{y})^2 \geq 0$$

$$(\sqrt{x} - \sqrt{y})(\sqrt{x} - \sqrt{y}) \geq 0$$

$$x - 2\sqrt{x}\sqrt{y} + y \geq 0$$

$$x - 2\sqrt{x}\sqrt{y} + y \geq 0$$

$$x + y \geq 2\sqrt{x}\sqrt{y}$$

$$2\sqrt{x}\sqrt{y} \leq x + y$$

$$\therefore \sqrt{x}\sqrt{y} \leq \frac{x+y}{2}$$

$$\therefore \boxed{\sqrt{xy} \leq \frac{x+y}{2}}$$

(b) If $x=-2$ and $y=-3$, then:

$LHS = \sqrt{(-2)(-3)} \ \underline{= \sqrt{6}}$

$RHS = \dfrac{-2+(-3)}{2} = \dfrac{-2-3}{2} = \underline{-\dfrac{5}{2}}$

so $\sqrt{xy} > \dfrac{x+y}{2}$ in this case

(Total for Question 11 is 3 marks)

12. A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$\underline{2^{2x} + 2^4} - 9(2^x) = 0$$

Let $\quad 2^x = y$

$$y^2 - 9y \underline{+ 8} = 0$$

$$(y - 8)(y - 1) = 0$$

$$y = 8 \text{ or } y = 1$$

So $x = 3$ or $x = 0$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

(a) $2^{2x+4} = 2^{2x} \cdot 2^4$ not $2^{2x} + 2^4$

Also, $2^4 = 16$ not 8

(b) $2^{2x+4} - 9(2^x) = 0$

$2^{2x} \cdot 2^4 - 9(2^x) = 0$

$16(2^{2x}) - 9(2^x) = 0$

Let $2^x = y$, then $16y^2 - 9y = 0$

$y(16y - 9) = 0$

Either $y = 0$ or $16y - 9 = 0$

$y = 0$ or $y = \frac{9}{16}$

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Question 12 continued

So, $2^x = 0$ or $2^x = \frac{9}{16}$

$2^x = 0$ is Invalid

Now, $2^x = \frac{9}{16}$

$$\boxed{x = \log_2\left(\frac{9}{16}\right)}$$

OR $x = \frac{\log\left(\frac{9}{16}\right)}{\log 2}$

(Total for Question 12 is 4 marks)

13. (a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

(a) $x^3+10x^2+25x = x(x^2+10x+25)$

$= x(x+5)(x+5)$

$= \boxed{x(x+5)^2}$

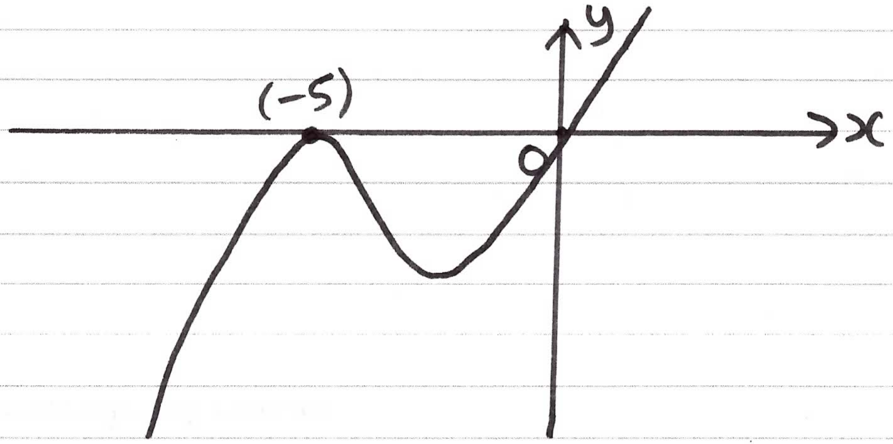
b) $y = x^3+10x^2+25x$

When the curve crosses the x-axis, $y=0$

$\therefore x^3+10x^2+25x = 0$

$x(x+5)^2 = 0$

Either $x=0$ or $x=-5$ twice



Question 13 continued

(c) Let $x^3+10x^2+25x=f(x)$

Then $(x+a)^3+10(x+a)^2+25(x+a)$ is a transformation horizontally of '$-a$'.

If $(-3,0)$ lies on the curve $f(x+a)$, then the original x-intercept of $(-5,0)$ has been translated ~~verti~~ horizontally by '$+2$' or the x-intercept of $(0,0)$ has been translated horizontally by '-3'.

$\therefore \boxed{a=-2 \text{ or } a=3}$

(Total for Question 13 is 7 marks)

14.

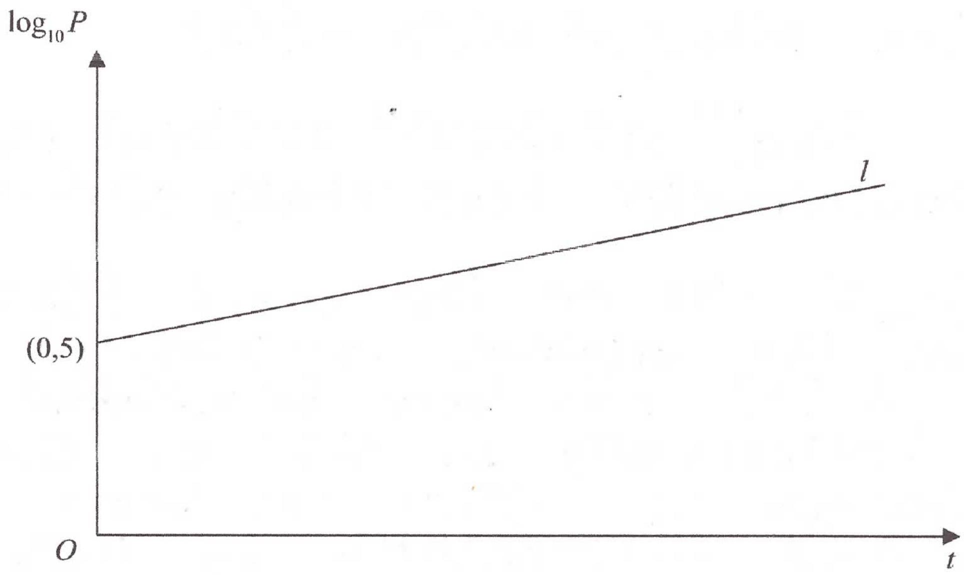


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

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Question 14 continued

(a) $y = mx + c \Rightarrow \log_{10} P = mt + c$

$$\boxed{\log_{10} P = \frac{1}{200} t + 5}$$

(b) As $P = ab^t$, then $\log_{10} P = \log_{10}(ab^t)$

$$\log_{10} P = \log_{10} a + \log_{10} b^t$$

$$\log_{10} P = \log_{10} a + t \log_{10} b$$

$$\log_{10} a = 5 \quad \text{and} \quad \log_{10} b = \frac{1}{200}$$

$$a = 10^5, \quad b = 10^{(1/200)}$$

$$\boxed{a = 100{,}000 \text{ and } b = 1.0116 \text{ (4s.f.)}}$$

(c) (i) a is the initial population

(ii) b is the proportional increase of population each year

(d) (i) $P = (100{,}000)(1.0116^{100})$

$= 316{,}227.766 = 300{,}000$ (nearest hundred thousand)

(ii) $200{,}000 = (100{,}000)(1.0116^t)$

$2 = 1.0116^t$

$t = \log_{(10^{\frac{1}{200}})}(2) \Rightarrow t = 60.2$ years (to 3s.f.)

(e) • The model predicts that growth never stops.

• 100 years is too far away to predict populations.

(Total for Question 14 is 13 marks)

15.

Diagram not drawn to scale

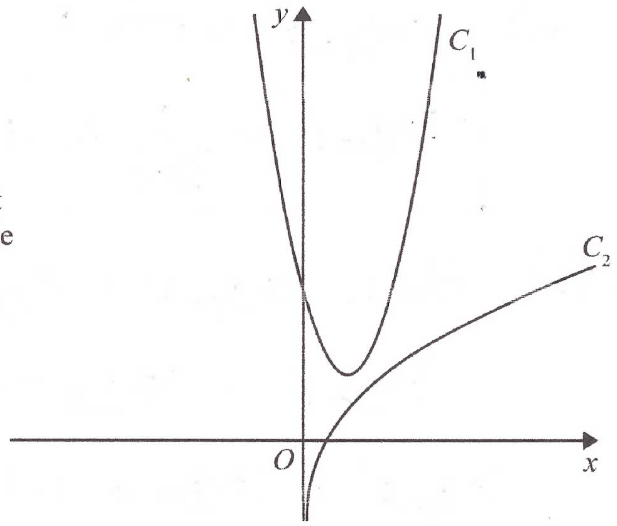

Figure 3

The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(8)

$y = 4x^2 - 6x + 4$

$\frac{dy}{dx} = 8x - 6$

As P lies on C_1, substitute in $x = \frac{1}{2}$ to find the gradient at P:

$\frac{dy}{dx} = 8\left(\frac{1}{2}\right) - 6$

$= 4 - 6$

$= -2$

Then, the gradient of the normal at P must be $\frac{1}{2}$

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Question 15 continued

Equation of normal at P: $y - y_1 = m(x - x_1)$

Using $P(\frac{1}{2}, 2)$ and $m = \frac{1}{2}$ $\rightarrow$ $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{1}{2}\left(x - \frac{1}{2}\right)$$

$$2y - 4 = x - \frac{1}{2}$$

$$y = \frac{1}{2}x + \frac{7}{4}$$

To find the point of intersection between the normal and C_2, solve simultaneous equations:

$y = \frac{1}{2}x + \frac{7}{4}$ (1) $\qquad$ $y = \frac{1}{2}x + \ln(2x)$ (2)

Substitute (1) into (2): $\frac{1}{2}x + \frac{7}{4} = \frac{1}{2}x + \ln(2x)$

$$\ln(2x) = \frac{7}{4}$$

$$2x = e^{7/4}$$

$$x = \frac{e^{7/4}}{2}$$

Substitute into (1) for y: $y = \frac{1}{2}\left(\frac{e^{7/4}}{2}\right) + \frac{7}{4}$

$$y = \frac{e^{7/4}}{4} + \frac{7}{4}$$

$\therefore$ coordinates of Q are $\boxed{\left(\frac{e^{7/4}}{2}, \frac{e^{7/4} + 7}{4}\right)}$

(Total for Question 15 is 8 marks)

16.

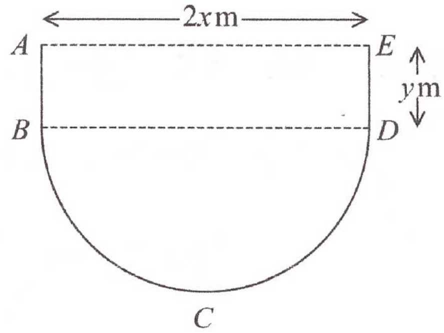

Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is $250\,\text{m}^2$,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

(a) $A = 250 = 2xy + \frac{\pi(x)^2}{2}$

$250 = 2xy + \frac{\pi(x^2)}{2}$

$250 = 2xy + \frac{\pi x^2}{2}$

So $y = \frac{\left(250 - \frac{\pi x^2}{2}\right)}{2x}$

Now, $P = 2x + 2y + \frac{2\pi r}{2} = 2x + 2y + \pi r$

$= 2x + 2y + \pi x$

Question 16 continued

Substitute y as $\dfrac{\left(250-\frac{\pi x^2}{2}\right)}{2x}$:

$$P = 2x + \cancel{2}\left(\frac{250-\frac{\pi x^2}{2}}{\cancel{2}x}\right) + \pi x$$

$$P = 2x + \frac{250}{x} - \frac{\pi x^2}{2x} + \pi x$$

$$P = 2x + \frac{250}{x} + \pi x - \frac{\pi x^2}{2x}$$

$$P = 2x + \frac{250}{x} + \frac{2\pi x^2 - \pi x^2}{2x}$$

$$P = 2x + \frac{250}{x} + \frac{\pi x^{\cancel{2}}}{2\cancel{x}}$$

$$\therefore \boxed{P = 2x + \frac{250}{x} + \frac{\pi x}{2}}$$

(b) $x > 0$ and $y > 0$ since both are lengths

$x > 0$ and $\dfrac{\left(250-\frac{\pi x^2}{2}\right)}{2x} > 0$

$$250 - \frac{\pi x^2}{2} > 0$$

(Total for Question 16 is 10 marks)

Question 16 continued

$$250 > \frac{\pi x^2}{2}$$

$$500 > \pi x^2$$

$$\frac{500}{\pi} > x^2$$

$$x^2 < \frac{500}{\pi}$$

$$\therefore \quad x < \sqrt{\frac{500}{\pi}}$$

And together with $x > 0$, $\boxed{0 < x < \sqrt{\frac{500}{\pi}}}$

(c) $P = 2x + \frac{250}{x} + \frac{\pi x}{2}$

$$P = 2x + 250x^{-1} + \frac{\pi}{2}x$$

$$\frac{dP}{dx} = 2 - 250x^{-2} + \frac{\pi}{2}$$

$$= 2 - \frac{250}{x^2} + \frac{\pi}{2}$$

At minimum, $\frac{dP}{dx} = 0$, so $2 - \frac{250}{x^2} + \frac{\pi}{2} = 0$

(Total for Question 16 is 10 marks)

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Question 16 continued

$$2+\frac{\pi}{2} = \frac{250}{x^2}$$

$$2x^2 + \frac{\pi}{2}x^2 = 250$$

$$\left(2+\frac{\pi}{2}\right)x^2 = 250$$

$$x^2 = \frac{250}{\left(2+\frac{\pi}{2}\right)}$$

$$x^2 = 70.012\ldots$$

$$\underline{x = 8.36\text{ m (to 3s.f.)}}$$

Substitute x into $P = 2x + \frac{250}{x} + \frac{\pi x}{2}$

$$P = 2(8.36) + \frac{250}{8.36} + \frac{\pi(8.36)}{2}$$

$$P = 59.744\ldots$$

$$\boxed{P = 59.8\text{m (to 3s.f.)}}$$

(Total for Question 16 is 10 marks)

17. A circle C with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle C also passes through the point $(10, 1)$.

(3)

The tangent to the circle C at the point $(10, 11)$ meets the y axis at the point P and the tangent to the circle C at the point $(10, 1)$ meets the y axis at the point Q.

(b) Show that the distance PQ is 58 explaining your method clearly.

(7)

(a) Radius of circle = distance between $(-2,6)$ and $(10,11)$

$$\text{Radius} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2-10)^2+(6-11)^2}$$
$$= \sqrt{(-12)^2+(-5)^2}$$
$$= \sqrt{144+25}$$
$$= \sqrt{169}$$
$$= 13 \text{ units}$$

Now, distance between $(-2,6)$ and $(10,1)$:

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-2-10)^2+(6-1)^2}$$
$$= \sqrt{(-12)^2+5^2}$$
$$= \sqrt{144+25}$$
$$= \sqrt{169}$$
$$= 13 \text{ units}$$

Distance are equal, and so $(10,1)$ lies on the circle

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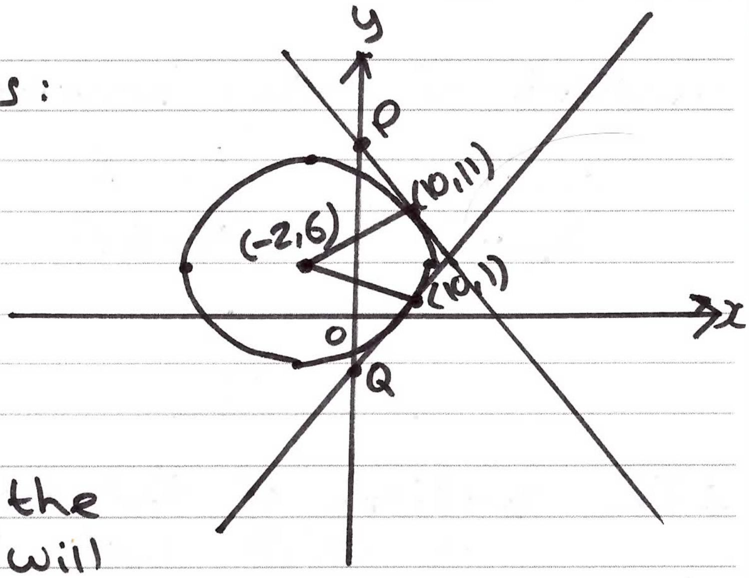
Question 17 continued

(b) Gradient of radius:

$$\frac{y_2-y_1}{x_2-x_1} = \frac{11-6}{10-(-2)}$$

$$= \frac{5}{12}$$



∴ the gradient of the tangent at (10,11) will be $-\frac{12}{5}$

Equation of tangent at (10,11):

$$y-y_1=m(x-x_1)$$

$$y-11=\frac{-12}{5}(x-10)$$

$$5(y-11)=-12(x-10)$$

$$5y-55=-12x+120$$

$$12x+5y-175=0$$

When this line cuts the y-axis, $x=0$

∴ $5y-175=0$

$5y=175 \Rightarrow y=35$

∴ P is at (0,35)

Question 17 continued

Gradient of radius between centre and (10,1):

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-6}{10-(-2)}$$

$$= \frac{-5}{12}$$

$\therefore$ the gradient of the tangent at (10,1) will be $\frac{12}{5}$.

Equation of tangent at (10,1):

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{12}{5}(x - 10)$$

$$5(y-1) = 12(x-10)$$

$$5y - 5 = 12x - 120$$

$$12x - 5y - 115 = 0$$

When this line cuts the y-axis, $x = 0$

$\therefore -5y = 115 \Rightarrow y = -23$

$\therefore$ Q is at $(0, -23)$

Distance PQ $= 35 + 23 \boxed{= 58}$

(Total for Question 17 is 10 marks)

TOTAL FOR PAPER IS 100 MARKS

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